The procedure used to establish design criteria for band-stop filters utilizing the resonators of Fig. 3 is straightforward. The basic section of the filter of Fig. 1 is considered to be a quarter-wavelength transmission line shunted at its mid-point by a series resonant circuit as shown in Fig. 4. The image parameters of the resonators of Figs. 3 and 4 are then equated. The slope parameter of the shunt series-resonant circuit is then related to the coupling parameter of the parallel coupled lines.

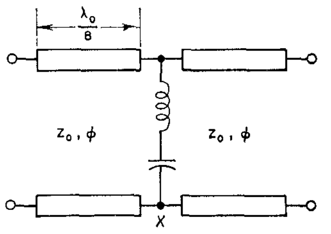

Fig. 4—Basic section of filter in Fig. 1.

For the resonator of Fig. 3 the image parameters are

$$Z_I = Z_0\sqrt{\frac{\tan^2\phi - k}{k\tan^2\phi - 1}}, \qquad (1)$$

$$\gamma_I = 2\tanh^{-1}\cdot\left\{\tan\phi\left[\frac{1 - k\tan^2\phi}{\tan^2\phi - k}\right]^{1/2}\right\}, \qquad (2)$$

where

$$k = \sqrt{1-c^2} \simeq 1 - c^2/2 \quad \text{for } c^2 \ll 1, \qquad (3)$$

$$c = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} = \text{coupling parameter}, \qquad (4)$$

Z_{oe} = even-mode characteristic impedance with respect to ground of each conductor.

Z_{oo} = odd-mode characteristic impedance with respect to ground of each conductor.

ϕ is defined in Fig. 3.

For the resonator of Fig. 4 the image parameters are

$$Z_I' = Z_0\sqrt{\frac{\tan\phi(\tan\phi + y)}{y\tan\phi - 1}}, \qquad (5)$$

$$\gamma_I' = 2\tanh^{-1}\tan\phi\cdot\left[\frac{1 - y\tan\phi}{\tan\phi(\tan\phi + y)}\right]^{1/2}, \qquad (6)$$

where

$$y = \frac{2}{Z_0}\left(\omega L - \frac{1}{\omega C}\right). \qquad (7)$$

Thus if $Z_I = Z_I'$, then $\gamma_I = \gamma_I'$. Equating the image impedances yields

$$y = \frac{2 - c^2}{c^2}(1 - \cot^2\phi). \qquad (8)$$

The slope parameter of the shunt series-resonant circuit is given by

$$x = \frac{\omega_0}{2}\left.\frac{dX}{d\omega}\right|_{\omega=\omega_0} \qquad (9)$$

and is given in terms of the low-pass prototype elements by Young, *et al.* Using (7) and (8)

$$\left.\frac{dy}{d\omega}\right|_{\omega=\omega_0} = \frac{4}{\omega_0}\frac{x}{Z_0} \qquad (10)$$

$$= \frac{2 - c^2}{c^2}\frac{\pi}{\omega_0}. \qquad (11)$$

Therefore,

$$\frac{c^2}{2 - c^2} = \frac{\pi}{4}\left(\frac{Z_0}{x}\right)$$

or

$$c^2 = \frac{\frac{\pi}{2}\left(\frac{Z_0}{x}\right)}{1 + \frac{\pi}{4}\frac{Z_0}{x}}. \qquad (12)$$

Using the expressions for x_i, the slope parameter for the ith shunt resonator, given by Young,

$$c_1 = \sqrt{\frac{2\pi\omega_1' g_o g_1}{4w^{-1} + \pi\omega_1' g_o g_1}}; \qquad (13)$$

$$c_i = \sqrt{\frac{2\pi\omega_1' g_i}{4g_o w^{-1} + \pi\omega_1' g_i}} \quad \text{for } i\text{-even}, \qquad (14)$$

$$c_i = \sqrt{\frac{2\pi\omega_1' g_o g_i}{4w^{-1} + \pi\omega_1' g_i g_o}} \quad \text{for } i\text{-odd}. \qquad (15)$$

This completes the derivation of the design criteria. Identical results are obtained by considering the behavior of the resonators near resonance, using the expressions for L and C given by Young, *et al.*, in terms of the low pass prototype, and satisfying (8). In this case the approximations required are

$$\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) \simeq 2\left(\frac{\omega - \omega_0}{\omega_0}\right) \qquad (16)$$

and

$$\left[1 + \frac{\pi}{4}\left(\frac{\omega - \omega_0}{\omega_0}\right)\right]^2 \simeq 1 + \frac{\pi}{2}\left(\frac{\omega - \omega_0}{\omega_0}\right). \qquad (17)$$

These approximations are quite valid for narrow bandwidth filters.

It should also be noted that a second type of resonator similar to that shown in Fig. 3 may also be used. For the latter the short circuits in Fig. 3 are replaced by open circuits. The design criteria may be developed in a manner similar to the above. However, this type of resonator requires compensation due to fringing at the open circuit ends. In addition some sort of support such as a dielectric post is required which tends to decrease unloaded Q. Hence, it is believed that the resonator of Fig. 3 is more suitable for practical applications.

Model work on filters of this type will begin shortly.

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Comments on "Maximum Efficiency of a Two Arm Waveguide Junction"*

In connection with a recent communication by Beatty[1] I wish to advise you that we have been studying the two-arm dissipative waveguide junction in our laboratory. Two years ago I published[2,3] a new demonstration of Deschamp's method for measuring scattering coefficients and the general properties of those coefficients.

In a recent work, not yet published, we show the variations of the absorbed power against the reflection coefficient of the terminal load.

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* Received July 22, 1963.

[1] R. W. Beatty, "Maximum efficiency of a two arm waveguide junction," IEEE TRANS. ON MICROWAVE THEORY AND TECHNIQUES (*Correspondence*), vol. MTT-11, p. 94; January, 1963.

[2] S. Lefeuvre, "Détermination de l'impédance caractéristique d'un quadripole quelconque en hyperfréquences," *Compt. Rend. Acad. Sci.*, vol. 250, pp. 3288–3289, 1960.

[3] S. Lefeuvre, "Quelque propriétés des quadripoles dissipatifs en hyperfréquences," *Compt. Rend. Acad. Sci.*, vol. 252, pp. 4135–4136; 1961.

A Uniform Coaxial Line with an Elliptic-Circular Cross Section*

Analysis and design of a nonuniform coaxial line with an isoperimetric sheath deformation has been reported.[1] The object of this note is to show that the procedure followed therein can be adopted for evaluating some of the essential features of an infinitely long ideal transmission line with an elliptic sheath and a circular inner conductor. Apart from its reported use with the nonuniform line, this type of structure may also be found in medium and large sized electronuclear machines.[2]

I. LINE CONSTANTS

Eqs. (15)–(21) in the communication quoted[1] provide expressions for the primary and the secondary constants of the line as follows:

$$\text{Capacitance per unit length} = C = \frac{4\pi\epsilon}{G} \qquad (1a)$$

External inductance per unit length

$$= l^e = \frac{\mu G}{4\pi} \qquad (1b)$$

Characteristic impedance

$$= Z_0 = \frac{G}{4\pi}\sqrt{\frac{\mu}{\epsilon}} \qquad (2a)$$

* Received March 11, 1963; revised manuscript received July 23, 1963.

[1] N. Seshagiri, "A non-uniform line with an isoperimetric sheath deformation," this issue, page 478.

[2] P. M. Morse and H. Feshbach, "Methods of Theoretical Physics," McGraw-Hill Book Co., Inc., New York, N. Y., p. 1204; 1953.